On functorial equivalence classes of blocks of finite groups

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Abstract

Let k be an algebraically closed field of characteristic $p > 0$ and let $\mathbb{F}$ be an algebraically closed field of characteristic 0. Recently, together with Bouc, we introduced the notion of functorial equivalences between blocks of finite groups and proved that given a p-group D, there is only a finite number of pairs (G, b) of a finite group G and a block b of kG with defect groups isomorphic to D, up to functorial equivalence over $\mathbb{F}$. In this paper, we classify the functorial equivalence classes over $\mathbb{F}$ of blocks with cyclic defect groups and 2-blocks of defects 2 and 3. In particular, we prove that for all these blocks, the functorial equivalence classes depend only on the fusion system of the block.

Keywords: block, functorial equivalence, fusion system, splendid Rickard equivalence
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1 Introduction

Throughout of the paper, k denotes an algebraically closed field of characteristic $p > 0$ and $\mathbb{F}$ denotes an algebraically closed field of characteristic zero. The local-global phenomena in modular representation theory of finite groups asserts that the global invariants of blocks are determined by the local invariants. There are many outstanding conjectures that revolves around this principle. One such conjecture is Puig's finiteness conjecture which asserts that given a finite p-group D, there are only finitely many pairs (G, b) of a finite group G and a block idempotent b of kG with defect group D, up to splendid Morita equivalence (Conjecture 6.4.2 in [L18]). Splendidly Morita equivalent blocks have isomorphic source algebras, and hence Puig's conjecture, if true, means that all the global invariants of a block are determined by the defect group up to finitely many possibilities.

In [BY22], together with Bouc, we introduced the notion of functorial equivalences over $\mathbb{F}$ between blocks of finite groups, weaker than splendid Morita equivalence, and proved the following finiteness theorem.

1.1 Theorem *[BY22, Theorem 10.6] Given a finite p-group D, there is only a finite number of pairs (G, b), where G is a finite group and b is a block idempotent of kG with defect group D, up to functorial equivalence over $\mathbb{F}$.*

To prove Puig's conjecture, it suffices to show that for a given p-group D every functorial equivalence class of blocks with defect D is a union of finitely many splendid Morita equivalence classes. Therefore, it is a natural question to classify the functorial equivalence classes of blocks with a given defect group D. In this paper, we start the program of classifying the functorial equivalence classes of blocks and consider the cases where $D \in \{C_{p^n}, V_4, Q_8, D_8, C_2 \times C_2 \times C_2, C_2 \times C_4\}$. We summarize our result as follows. For a finite group G we denote by $b_0(G)$ the principal block of kG.

1.2 Theorem *Let G be a finite group and let b be a block idempotent of kG with a defect group D.*

(a) *The functorial equivalence classes over $\mathbb{F}$ of blocks with cyclic defect groups depend only on the inertial quotient of the blocks. In particular, for blocks with cyclic defect groups the functorial equivalence classes over $\mathbb{F}$ coincide with the splendid Rickard equivalence classes.*

(b) *If $D = V_4$, then the pair (G, b) is functorially equivalent over $\mathbb{F}$ to either $(V_4, 1)$ or $(A_4, 1)$. In particular, for blocks with Klein four defect groups the functorial equivalence classes over $\mathbb{F}$ coincide with the splendid Rickard equivalence classes.*

(c) *If $D = Q_8$, then the pair (G, b) is functorially equivalent over $\mathbb{F}$ to either $(Q_8, 1)$ or $(SL(2,3), b_0(SL(2,3))$.*

(d) *If $D = D_8$, then then the pair (G, b) is functorially equivalent over $\mathbb{F}$ to $(D_8, 1)$, $(S_4, b_0(S_4))$ or $(PSL(3,2), b_0(PSL(3,2))$.*

(e) *If $D = C_2 \times C_2 \times C_2$, then the pair (G, b) is functorially equivalent over $\mathbb{F}$ to $(C_2 \times C_2 \times C_2, 1)$, $(A_4 \times C_2, 1)$, $(SL_2(8), b_0(SL_2(8)))$ or $(J_1, b_0(J_1))$. In particular, for bloks with defect groups $C_2 \times C_2 \times C_2$ the functorial equivalence classes over $\mathbb{F}$ coincide with the isotypy classes.*

(f) *If $D = C_2 \times C_4$, then the pair (G, b) is functorially equivalent over $\mathbb{F}$ to $(C_2 \times C_4, 1)$.*

Theorem 1.2 follows from the more precise Theorems 3.1, 4.1, 5.1, 6.1, 7.1 and 8.1. The following corollary is immediate from Theorem 1.2.

1.3 Corollary *Functorial equivalence classes over $\mathbb{F}$ of blocks of finite groups with defect groups $D \in \{C_{p^n}, V_4, Q_8, D_8, C_2 \times C_2 \times C_2, C_2 \times C_4\}$ depend only on the fusion system of the blocks.*

We also find the composition factors of the diagonal p-permutation functors arising from all these blocks except when $D = C_2 \times C_2 \times C_2$.

In Section 2 we recall diagonal p-permutation functors and functorial equivalences of blocks. We consider blocks with cyclic defect groups in Section 3, with Klein four defect groups in Section 4, with Q_8 defect groups in Section 5, with D_8 defect groups in Section 6, with $C_2 \times C_2 \times C_2$ defect groups in Section 7 and with $C_2 \times C_4$ defect groups in Section 8.

2 Preliminaries

(a) Let (P, s) be a pair where P is a p-group and s is a generator of a p'-group acting on P. We write $P\langle s\rangle := P \rtimes \langle s\rangle$ for the corresponding semi-direct product. We say that two pairs (P, s) and (Q, t) are *isomorphic* and write $(P, s) \cong (Q, t)$, if there is a group isomorphism $f : P\langle s\rangle \to Q\langle t\rangle$ that sends s to a conjugate of t. We set $\mathrm{Aut}(P, s)$ to be the group of automorphisms of the pair (P, s) and $\mathrm{Out}(P, s) = \mathrm{Aut}(P, s)/\mathrm{Inn}(P\langle s\rangle)$. Recall from [BY20] that a pair (P, s) is called a *D^Δ-pair*, if $C_{\langle s\rangle}(P) = 1$. See also [BY22, Lemma 6.10].

(b) Let G, H and K be finite groups. We call a (kG, kH)-bimodule M a *diagonal p-permutation bimodule*, if M is a p-permutation (kG, kH)-bimodule whose indecomposable direct summands have twisted diagonal vertices as subgroups of $G \times H$, or equivalently, if M is a p-permutation (kG, kH)-bimodule which is projective both as a left kG-module and as a right kH-module. We denote by $T^\Delta(kG, kH)$ the Grothendieck group of diagonal p-permutation (kG, kH)-bimodules. For a commutative ring R, we also set $RT^\Delta(kG, kH) := R \otimes_{\mathbb{Z}} T^\Delta(kG, kH)$. If b is a block idempotent of kG and c a block idempotent of kH, then we define $T^\Delta(kGb, kHc)$ and $RT^\Delta(kGb, kHc)$ similarly.

If M is a diagonal p-permutation (kG, kH)-bimodule and N a diagonal p-permutation (kH, kK)-bimodule, then the tensor product $M \otimes_{kH} N$ is a diagonal p-permutation (kG, kK)-bimodule. This induces an R-linear map

$$\cdot_H : RT^\Delta(kG, kH) \times RT^\Delta(kH, kK) \to RT^\Delta(kG, kK)\,.$$

(c) Let Rpp_k^Δ denote the following category:

- objects: finite groups.
- $\mathrm{Mor}_{Rpp_k^\Delta}(G, H) = RT^\Delta(kH, kG)$.
- composition is induced from the tensor product of bimodules.
- $\mathrm{Id}_G = [kG]$.

An R-linear functor from Rpp_k^Δ to ${}_R\mathsf{Mod}$ is called a *diagonal p-permutation functor* over R. Together with natural transformations, diagonal p-permutation functors form an abelian category $\mathcal{F}^\Delta_{Rpp_k}$.

(d) Let G be a finite group and b a block idempotent of kG. Recall from [BY22] that the block diagonal p-permutation functor $RT^\Delta_{G,b}$ is defined as

$$\begin{aligned} RT^\Delta_{G,b} : Rpp_k^\Delta &\to {}_R\mathsf{Mod} \\ H &\mapsto RT^\Delta(kH, kG) \otimes_{kG} kGb\,. \end{aligned}$$

If H is another finite group and if c is a block idempotent of kH, we say that the pairs (G, b) and (H, c) are *functorially equivalent* over R, if the corresponding diagonal p-permutation functors $RT^\Delta_{G,b}$ and $RT^\Delta_{H,c}$ are isomorphic in $\mathcal{F}^\Delta_{Rpp_k}$ ([BY22, Definition 10.1]). By [BY22, Lemma 10.2] the pairs (G, b) and (H, c) are functorially equivalent over R if and only if there exist $\omega \in RT^\Delta(kGb, kHc)$ and $\sigma \in RT^\Delta(kHc, kGb)$ such that

$$\omega \cdot_G \sigma = [kGb] \quad \text{in} \quad RT^\Delta(kGb, kGb) \quad \text{and} \quad \sigma \cdot_H \omega = [kHc] \quad \text{in} \quad RT^\Delta(kHc, kHc)\,.$$

Note that this implies that a p-permutation equivalence between blocks implies a functorial equivalence over $\mathbb{Z}$ and hence a functorial equivalence over R, for any R.

(e) Recall from [BY22] that the category $\mathcal{F}^\Delta_{\mathbb{F}pp_k}$ is semisimple. Moreover, the simple diagonal p-permutation functors $S_{L,u,V}$, up to isomorphism, are parametrized by the isomorphism classes of triples (L, u, V) where (L, u) is a D^Δ-pair, and V is a simple $\mathbb{F}\mathrm{Out}(L, u)$-module (see [BY22, Sections 6 and 7] for more details on simple functors).

(f) Since the category $\mathcal{F}^\Delta_{\mathbb{F}pp_k}$ is semisimple, the functor $\mathbb{F}T^\Delta_{G,b}$ is a direct sum of simple diagonal p-permutation functors $S_{L,u,V}$. Hence two pairs (G, b) and (H, c) are functorially equivalent over $\mathbb{F}$ if and only if for any triple (L, u, V), the multiplicities of the simple diagonal p-permutation functor $S_{L,u,V}$ in $\mathbb{F}T^\Delta_{G,b}$ and $\mathbb{F}T^\Delta_{H,c}$ are the same. We now recall the formula for the multiplicity of $S_{L,u,V}$ in $\mathbb{F}T^\Delta_{G,b}$. See [BY22, Section 8] for more details.

Let (D, e_D) be a maximal kGb-Brauer pair. For any subgroup $P \leqslant D$, let e_P be the unique block idempotent of $kC_G(P)$ with $(P, e_P) \leqslant (D, e_D)$ (see, for instance, [L18, Section 6.3] for more details on Brauer pairs). Let also $\mathcal{F}_b$ be the fusion system of kGb with respect to (D, e_D) and let $[\mathcal{F}_b]$ be a set of isomorphism classes of objects in $\mathcal{F}_b$.

For $P \in \mathcal{F}_b$, we set $\mathcal{P}_{(P,e_P)}(L, u)$ to be the set of group isomorphisms $\pi : L \to P$ with $\pi i_u \pi^{-1} \in \mathrm{Aut}_{\mathcal{F}_b}(P)$. The set $\mathcal{P}_{(P,e_P)}(L, u)$ is an $(N_G(P, e_P), \mathrm{Aut}(L, u))$-biset via

$$g \cdot \pi \cdot \varphi = i_g \pi \varphi$$

for $g \in N_G(P, e_P)$, $\pi \in \mathcal{P}_{(P,e_P)}(L, u)$ and $\varphi \in \mathrm{Aut}(L, u)$. We denote by $[\mathcal{P}_{(P,e_P)}(L, u)]$ a set of representatives of $N_G(P, e_P) \times \mathrm{Aut}(L, u)$-orbits of $\mathcal{P}_{(P,e_P)}(L, u)$.

For $\pi \in [\mathcal{P}_{(P,e_P)}(L, u)]$, the stabilizer in $\mathrm{Aut}(L, u)$ of the $N_G(P, e_P)$-orbit of π is denoted by $\mathrm{Aut}(L, u)_{\overline{(P,e_P,\pi)}}$. One has

$$\mathrm{Aut}(L, u)_{\overline{(P,e_P,\pi)}} = \{\varphi \in \mathrm{Aut}(L, u) \mid \pi\varphi\pi^{-1} \in \mathrm{Aut}_{\mathcal{F}_b}(P)\}\,.$$

2.1 Theorem *[BY22, Theorem 8.22(b)] The multiplicity of a simple diagonal p-permutation functor $S_{L,u,V}$ in the functor $\mathbb{F}T^{\Delta}_{G,b}$ is equal to the $\mathbb{F}$-dimension of*

$$\bigoplus_{P\in[\mathcal{F}_b]} \bigoplus_{\pi\in[\mathcal{P}_{(P,e_P)}(L,u)]} \mathbb{F}\mathrm{Proj}(ke_PC_G(P), u) \otimes_{\mathrm{Aut}(L,u)_{\overline{(P,e_P,\pi)}}} V\,.$$

Let G be a finite group. We denote by $\mathcal{Q}_{G,p}$ the set of pairs (P, s) where P is a p-subgroup of G and s is a p'-element of $N_G(P)$. The group G acts on $\mathcal{Q}_{G,p}$ via conjugation and we denote by $[\mathcal{Q}_{G,p}]$ a set of representatives of the G-orbits on $\mathcal{Q}_{G,p}$.

If $(P, s) \in \mathcal{Q}_{G,p}$, then the pair $(\tilde{P}, \tilde{s}) := (PC_{\langle s\rangle}(P)/C_{\langle s\rangle}(P), sC_{\langle s\rangle}(P))$ is a D^{Δ}-pair. Suppose that (L, u) is another D^{Δ}-pair isomorphic to $(\tilde{P}, \tilde{s})$. Then the isomorphism between the pairs induces a group homomorphism from $N_G(P, s)$ to $\mathrm{Out}(L, u)$, see [BY22, Section 7]. So, a simple $\mathbb{F}\mathrm{Out}(L, u)$-module V can be viewed as an $\mathbb{F}N_G(P, s)$-module via this homomorphism.

2.2 Theorem *[BY22, Corollary 7.4] The multiplicity of a simple diagonal p-permutation functor $S_{L,u,V}$ in the representable functor $\mathbb{F}T^{\Delta}_{G}$ is equal to the $\mathbb{F}$-dimension of*

$$\bigoplus_{\substack{(P,s)\in[\mathcal{Q}_{G,p}]\\ (\tilde{P},\tilde{s})\cong(L,u)}} V^{N_G(P,s)}\,.$$

2.3 Notation Let G be a finite group and let b be a block idempotent of kG.

(a) We denote the multiplicity of a simple diagonal p-permutation functor $S_{L,u,V}$ in $\mathbb{F}T^{\Delta}_{G,b}$ by $\mathrm{Mult}(S_{L,u,V}, \mathbb{F}T^{\Delta}_{G,b})$.

(b) We denote by $l(kGb)$ the number of isomorphism classes of simple kGb-modules. By [BY22, Corollary 8.23], one has $\mathrm{Mult}(S_{1,1,\mathbb{F}}, \mathbb{F}T^{\Delta}_{G,b}) = l(kGb)$.

The following lemma will be used in Sections 5 and 6.

2.4 Lemma *Let G be a finite group and let b be a block idempotent of kG with a defect group D. Let (D, e_D) be a maximal b-Brauer pair and let $\mathcal{F}_b$ be the fusion system of b with respect to (D, e_D).*

Let $\overline{\mathrm{Aut}_{\mathcal{F}_b}(D)}$ denote the image of $\mathrm{Aut}_{\mathcal{F}_b}(D)$ in $\mathrm{Out}(D)$. Then for any simple $\mathbb{F}\mathrm{Out}(D)$-module V, we have

$$\mathrm{Mult}(S_{D,1,V}, \mathbb{F}T^{\Delta}_{G,b}) = \dim_{\mathbb{F}}\left(V^{\overline{\mathrm{Aut}_{\mathcal{F}_b}(D)}}\right).$$

Proof One shows that

$$\mathcal{P}_{(D,e_D)}(D,1) = \mathrm{Aut}(D) \quad \text{and} \quad [N_G(D,e_D)\backslash\mathcal{P}_{(D,e_D)}(D,1)/\mathrm{Aut}(D)] = [\mathrm{id}_D].$$

Moreover,

$$\mathrm{Aut}(D)_{\overline{(D,e_D,\mathrm{id}_D)}} = \mathrm{Aut}_{\mathcal{F}_b}(D).$$

Since $kC_G(D)e_D$ has a central defect group $Z(D)$, it has a unique isomorphism class of simple modules and hence

$$\mathbb{F}\mathrm{Proj}(kC_G(D)e_D, 1) \cong \mathbb{F}.$$

Theorem 2.1 implies now that

$$\mathrm{Mult}(S_{D,1,V}, \mathbb{F}T^{\Delta}_{G,b}) = \dim_{\mathbb{F}}\left(\mathbb{F}\otimes_{\mathrm{Aut}_{\mathcal{F}_b}(D)} V\right) = \dim_{\mathbb{F}}\left(V^{\mathrm{Aut}_{\mathcal{F}_b}(D)}\right) = \dim_{\mathbb{F}}\left(V^{\overline{\mathrm{Aut}_{\mathcal{F}_b}(D)}}\right),$$

as desired. □

3 Blocks with cyclic defect groups

Let G be a finite group and let b be a block idempotent of kG with a cyclic defect group D. We will give a decomposition of the functor $\mathbb{F}T^{\Delta}_{G,b}$ in terms of the simple diagonal p-permutation functors. We refer the reader to [L18, Chapter 11] for more details on blocks with cyclic defect groups. Let (D,e_D) be a maximal b-Brauer pair and let $E = N_G(D,e_D)/D$ be the inertial quotient of b. Then for every b-Brauer pair $(P,e_P) \leqslant (D,e_D)$ one has $N_G(P,e_P)/C_G(P) \cong E$, see, for instance, [L18, Theorem 11.2.1].

First of all, the multiplicity of $S_{1,1,\mathbb{F}}$ is equal to $l(kGb)$ which is equal to $|E|$ by [L18, Theorem 11.1.3]. Assume now that L is a nontrivial cyclic p-group. Then $\mathrm{Aut}(L)$ is an abelian group and hence one can show that for p'-elements $u, u' \in \mathrm{Aut}(L)$, the pairs (L,u) and (L,u') are isomorphic if and only if $u = u'$. Moreover, $\mathrm{Out}(L,u) \cong \mathrm{Aut}(L)/\langle u\rangle$ is abelian.

Let $P \leqslant D$ with $P \cong L$. We identify L with P and E with its image in $\mathrm{Aut}(P)$ under the map $E \to \mathrm{Aut}(P)$, $s \mapsto i_s$. Via these identifications we have $E = \mathrm{Aut}_{\mathcal{F}_b}(P)$.

For any p'-element $u \in \mathrm{Aut}(P)$, we have

$$\mathcal{P}_{(P,e_P)}(P,u) = \{\pi \in \mathrm{Aut}(P) \,|\, \pi i_u \pi^{-1} = i_u \in \mathrm{Aut}_{\mathcal{F}_b}(P)\} = \begin{cases} \mathrm{Aut}(P), & \text{if } u \in E \\ \emptyset, & \text{otherwise}. \end{cases}$$

If $u \notin E$, then the simple functor $S_{L,u,V}$ is not a summand of $\mathbb{F}T^{\Delta}_{G,b}$. If $u \in E$, then $\mathcal{P}_{(P,e_P)}(P,u) = \mathrm{Aut}(P)$, and hence one can show that there is only one $N_G(P,e_P)\times\mathrm{Aut}(P,u)$-orbit of $\mathrm{Aut}(P)$, i.e., $[\mathcal{P}_{(P,e_P)}(P,u)] = [\mathrm{id}]$. Moreover, one has

$$\mathrm{Aut}(P,u)_{\overline{(P,e_P,\mathrm{id})}} = \{\phi \in \mathrm{Aut}(P,u) \,|\, \exists g \in N_G(P,e_P), i_g = \phi\} = E.$$

Now since b is a block with cyclic defect group, by [L18, Theorem 11.2.1] the block idempotent e_P of $kC_G(P)$ is nilpotent, and so it has a unique simple module, up to isomorphism. Therefore we have $\mathbb{F}\mathrm{Proj}(keC_G(P), u) \cong \mathbb{F}$, and it follows that the multiplicity of the simple functor $S_{P,u,V}$, for $u \in E$, is equal to the $\mathbb{F}$-dimension of the fixed points V^E. Since $\mathrm{Out}(P, u)$ is abelian, the dimension of V is equal to one and hence V^E is either zero or equal to V. We proved the following.

3.1 Theorem *Let G be a finite group and let b be a block idempotent of kG with a cyclic defect group D and inertial quotient E. Then*

$$\mathbb{F}T^{\Delta}_{G,b} \cong |E| S_{1,1,\mathbb{F}} \bigoplus_{1<P\leqslant D} \bigoplus_{u\in E} \bigoplus_{\substack{V\in_{\mathbb{F}[\mathrm{Out}(P,u)/E]}\mathsf{mod} \\ simple}} S_{P,u,V}\,.$$

3.2 Corollary *Let G and H be finite groups. Let b be a block idempotent of kG and c a block idempotent of kH with cyclic defect groups isomorphic to D. Then (G, b) and (H, c) are functorially equivalent over $\mathbb{F}$ if and only if the inertial quotients of b and c are isomorphic. In particular, kGb and kHc are splendidly Rickard equivalent if and only if (G, b) and (H, c) are functorially equivalent over $\mathbb{F}$.*

Proof The first assertion follows from Theorem 3.1, and the second assertion follows from the first one and [Ro98]. □

4 Blocks with Klein four defect groups

Let C_2 denote a cyclic group of order 2 and let V_4 denote a Klein-four group. Since $\mathrm{Aut}(C_2) = \{1\}$, the functor $S_{C_2,1,\mathbb{F}}$ is the unique simple functor, up to isomorphism, with parametrizing set (L, u, V) where $L \cong C_2$.

Let $u \in \mathrm{Aut}(V_4) \cong \mathrm{Sym}(3)$ be an element of order 3. One shows that a D^{Δ}-pair (L, u) with $L \cong V_4$ is isomorphic to either $(V_4, 1)$ or (V_4, u). One can also show that $\mathrm{Out}(V_4, u) = \{1\}$. Let $\mathbb{F}_-$ and V_2 denote a non-trivial one dimensional module and a two dimensional simple module of $\mathbb{F}\mathrm{Out}(V_4) \cong \mathbb{F}\mathrm{Sym}(3)$, respectively.

4.1 Theorem *Let b be a block idempotent of kG with defect groups isomorphic to V_4. Then one of the following occurs:*

(i) *The block idempotent b is nilpotent and (G, b) is functorially equivalent over $\mathbb{F}$ to $(V_4, 1)$. In this case, one has*

$$\mathbb{F}T^{\Delta}_{G,b} \cong S_{1,1,\mathbb{F}} \oplus 3S_{C_2,1,\mathbb{F}} \oplus S_{V_4,1,\mathbb{F}} \oplus S_{V_4,1,\mathbb{F}_-} \oplus 2S_{V_4,1,V_2}\,.$$

(ii) *The pair (G, b) is functorially equivalent over $\mathbb{F}$ to $(A_4, 1)$. In this case, one has*

$$\mathbb{F}T^{\Delta}_{G,b} \cong 3S_{1,1,\mathbb{F}} \oplus S_{C_2,1,\mathbb{F}} \oplus S_{V_4,1,\mathbb{F}} \oplus S_{V_4,1,\mathbb{F}_-1} \oplus 2S_{V_4,u,\mathbb{F}}\,.$$

In particular, the functorial equivalence class of (G, b) depends only on the inertial quotient of b.

Proof It is well-known that if b is a block idempotent of a finite group G with defect groups isomorphic to V_4, then kGb is splendidly Rickard equivalent to either kV_4 or kA_4. Indeed, by [CEKL12], kGb is splendidly Morita equivalent to kV_4, kA_4 or $kA_5b_0(A_5)$, and by [R96, Section 3] kA_4 and $kA_5b_0(A_5)$ are splendidly Rickard equivalent. It follows that (G,b) is functorially equivalent over $\mathbb{F}$ to either $(V_4,1)$ or $(A_4,1)$. One can find the multiplicities of the simple functors in $\mathbb{F}T^{\Delta}_{V_4}$ and $\mathbb{F}T^{\Delta}_{A_4}$ easily using Theorem 2.2. □

5 Blocks with Q_8 defect groups

Let C_4 denote a cyclic group of order 4. Since $\mathrm{Aut}(C_4)=\mathrm{Out}(C_4)\cong C_2$ is a 2-group, the functors $S_{C_4,1,\mathbb{F}}$ and $S_{C_4,1,\mathbb{F}_-}$ are the only simple functors, up to isomorphism, with a parametrizing set (L,u,V) with $L\cong C_4$, where $\mathbb{F}$ and $\mathbb{F}_-$ denote the trivial and the non-trivial simple $\mathbb{F}\mathrm{Out}(C_4)$-modules.

Let Q_8 be a quaternion group of order 8. Let $u\in\mathrm{Aut}(Q_8)\cong\mathrm{Sym}(4)$ be an element of order 3. One shows that a D^{Δ}-pair (L,u) with $L\cong Q_8$ is isomorphic to either $(Q_8,1)$ or (Q_8,u). One can also show that $\mathrm{Out}(Q_8,u)=\{1\}$. Indeed, one can show that $\mathrm{Aut}(Q_8\rtimes\langle u\rangle)\cong\mathrm{Sym}(4)$ and $\mathrm{Inn}(Q_8\rtimes\langle u\rangle)\cong\mathrm{Alt}(4)$. Since $Q_8\rtimes\langle u\rangle$ has two conjugacy classes of 3-elements, but only one automorphism class of 3-elements, it follows that $\mathrm{Aut}(Q_8,u)=\mathrm{Inn}(Q_8\rtimes\langle u\rangle)$ and hence $\mathrm{Out}(Q_8,u)=\{1\}$. This implies that the simple functors $S_{Q_8,1,\mathbb{F}}$, $S_{Q_8,1,\mathbb{F}_-}$, $S_{Q_8,1,V_2}$ and $S_{Q_8,u,\mathbb{F}}$ are the only simple functors, up to isomorphism, with parametrizing set (L,u,V) with $L\cong Q_8$, where $\mathbb{F}_-$ and V_2 denote the nontrivial one dimensional and the two dimensional simple $\mathbb{F}\mathrm{Out}(Q_8)\cong\mathbb{F}\mathrm{Sym}(3)$-modules, respectively.

5.1 Theorem *Let b be a block idempotent of kG with defect groups isomorphic to Q_8. Then one of the following occurs:*

(i) *The block idempotent b is nilpotent and (G,b) is functorially equivalent over $\mathbb{F}$ to $(Q_8,1)$. In this case, one has*

$$\mathbb{F}T^{\Delta}_{G,b}\cong S_{1,1,\mathbb{F}}\oplus S_{C_2,1,\mathbb{F}}\oplus 3S_{C_4,1,\mathbb{F}}\oplus S_{Q_8,1,\mathbb{F}}\oplus S_{Q_8,1,\mathbb{F}_{-1}}\oplus 2S_{Q_8,1,V_2}\,.$$

(ii) *The pair (G,b) is functorially equivalent over $\mathbb{F}$ to $(SL(2,3),b_0)$, where b_0 is the principal 2-block of $SL(2,3)$. In this case, one has*

$$\mathbb{F}T^{\Delta}_{G,b}\cong 3S_{1,1,\mathbb{F}}\oplus 3S_{C_2,1,\mathbb{F}}\oplus S_{C_4,1,\mathbb{F}}\oplus S_{Q_8,1,\mathbb{F}}\oplus S_{Q_8,u,\mathbb{F}}\,.$$

Proof Let (D,e_D) be a maximal b-Brauer pair and for any $P\leqslant D$, let (P,e_P) denote the unique b-Brauer pair with $(P,e_P)\leqslant(D,e_D)$. Let also $\mathcal{F}$ denote the fusion system of b with respect to (D,e_D). Up to isomorphism, there are two fusion systems on Q_8.

First, assume that $\mathcal{F}$ is isomorphic to the inner fusion system on Q_8. Then the block idempotent b is nilpotent and hence by [BY22, Theorem 9.2], (G,b) is functorially equivalent over $\mathbb{F}$ to $(Q_8,1)$. Using Theorem 2.2 one can easily show that

$$\mathbb{F}T^{\Delta}_{G,b}\cong\mathbb{F}T^{\Delta}_{Q_8}\cong S_{1,1,\mathbb{F}}\oplus S_{C_2,1,\mathbb{F}}\oplus 3S_{C_4,1,\mathbb{F}}\oplus S_{Q_8,1,\mathbb{F}}\oplus S_{Q_8,1,\mathbb{F}_{-1}}\oplus 2S_{Q_8,1,V_2}\,.$$

Now assume that $\mathcal{F}$ is isomorphic to the non-inner fusion system on Q_8. Let Z denote the center of D. By [O75, Theorem 3.17], one has $l(kGb)=l(kC_G(Z)e_Z)=3$. Thus, Theorem 2.1 implies that

$$\mathrm{Mult}(S_{1,1,\mathbb{F}},\mathbb{F}T^{\Delta}_{G,b})=\mathrm{Mult}(S_{C_2,1,\mathbb{F}},\mathbb{F}T^{\Delta}_{G,b})=3\,.$$

We now find the multiplicities of the simple functors $S_{C_4,1,\mathbb{F}}$ and $S_{C_4,1,\mathbb{F}_-}$ in $\mathbb{F}T^{\Delta}_{G,b}$. Let P be a subgroup of D isomorphic to C_4. Note that all subgroups of D of order 4 are $\mathcal{F}$-isomorphic. The block $kC_G(P)e_P$ has a cyclic defect group $C_D(P) = P$ and so it follows that it has a unique isomorphism class of simple modules. Indetify P with $L = C_4$. One has

$$\mathcal{P}_{(P,e_P)}(C_4, 1) = \mathrm{Aut}(C_4) = C_2$$

and

$$[N_G(P, e_P)\backslash\mathcal{P}_{(P,e_P)}(C_4, 1)/\mathrm{Aut}(C_4)] = [\mathrm{id}]\,.$$

It follows that $\mathrm{Aut}(C_4)_{\overline{(C_4,e_{C_4},\mathrm{id})}} = \mathrm{Aut}(C_4)$. These imply that

$$\mathrm{Mult}(S_{C_4,1,\mathbb{F}}, \mathbb{F}T^{\Delta}_{G,b}) = \dim_{\mathbb{F}} \mathbb{F}^{C_2} = 1$$

and that

$$\mathrm{Mult}(S_{C_4,1,\mathbb{F}_-}, \mathbb{F}T^{\Delta}_{G,b}) = \dim_{\mathbb{F}}(\mathbb{F}_-)^{C_2} = 0\,.$$

We finally consider the case $L = Q_8$. Since $\overline{\mathrm{Aut}_{\mathcal{F}_b}(D)} \cong \mathrm{Out}(Q_8) \cong \mathrm{Sym}(3)$, Lemma 2.4 implies that

$$\mathrm{Mult}(S_{Q_8,1,\mathbb{F}}, \mathbb{F}T^{\Delta}_{G,b}) = 1 \quad \text{and} \quad \mathrm{Mult}(S_{Q_8,1,\mathbb{F}_-}, \mathbb{F}T^{\Delta}_{G,b}) = \mathrm{Mult}(S_{Q_8,1,V_2}, \mathbb{F}T^{\Delta}_{G,b}) = 0\,.$$

By Theorem 2.1, the multiplicity of $S_{Q_8,u,\mathbb{F}}$ in $\mathbb{F}T^{\Delta}_{G,b}$ is equal to the cardinality of the set

$$\left[N_G(D, e_D)\backslash\mathcal{P}_{(D,e_D)}(Q_8, u)/\mathrm{Aut}(Q_8, u)\right]\,.$$

One shows that $\mathcal{P}_{(Q_8,e_{Q_8})}(Q_8, u) = \mathrm{Aut}(Q_8)$ and since $\mathrm{Aut}_{\mathcal{F}_b}(Q_8) \cong \mathrm{Aut}(Q_8)$, it follows that

$$\mathrm{Mult}(S_{Q_8,u,\mathbb{F}}, \mathbb{F}T^{\Delta}_{G,b}) = 1\,.$$

This completes the proof. □

6 Blocks with D_8 defect groups

Let D_8 be a dihedral group of order 8. Since $\mathrm{Aut}(D_8)$ is a 2-group and since $\mathrm{Out}(D) \cong C_2$, the functors $S_{D_8,1,\mathbb{F}}$ and $S_{D_8,1,\mathbb{F}_-}$ are the only simple functors, up to isomorphism, with parametrizing set (L, u, V) with $L \cong D_8$, where $\mathbb{F}$ and $\mathbb{F}_-$ denote the trivial and the nontrivial simple $\mathbb{F}\mathrm{Out}(D) \cong \mathbb{F}C_2$-modules, respectively.

6.1 Theorem *Let b be a block idempotent of kG with defect groups isomorphic to D_8. Then one of the following occurs:*

(i) *The fusion system of b is the inner fusion system on D_8. In this case, b is nilpotent and (G, b) is functorially equivalent over $\mathbb{F}$ to $(D_8, 1)$. We have*

$$\mathbb{F}T^{\Delta}_{G,b} \cong S_{1,1,\mathbb{F}} \oplus 3S_{C_2,1,\mathbb{F}} \oplus S_{C_4,1,\mathbb{F}} \oplus 2S_{V_4,1,\mathbb{F}} \oplus 2S_{V_4,1,V_2} \oplus S_{D_8,1,\mathbb{F}} \oplus S_{D_8,1,\mathbb{F}_{-1}}\,.$$

(ii) *The fusion system of b is the non-inner non-simple fusion system on D_8. In this case, (G,b) is functorially equivalent over $\mathbb{F}$ to (S_4,b_0), where b_0 is the principal 2-block of S_4. We have*

$$\mathbb{F}T^{\Delta}_{G,b} \cong 2S_{1,1,\mathbb{F}} \oplus 2S_{C_2,1,\mathbb{F}} \oplus S_{C_4,1,\mathbb{F}} \oplus 2S_{V_4,1,\mathbb{F}} \oplus S_{V_4,1,V_2} \oplus S_{V_4,u,\mathbb{F}} \oplus S_{D_8,1,\mathbb{F}} \oplus S_{D_8,1,\mathbb{F}_{-1}}\,.$$

(iii) *The fusion system of b is the simple fusion system on D_8. In this case, (G,b) is functorially equivalent over $\mathbb{F}$ to $(PSL(3,2),b_0)$, where b_0 is the principal 2-block of $PSL(3,2)$. We have*

$$\mathbb{F}T^{\Delta}_{G,b} \cong 3S_{1,1,\mathbb{F}} \oplus S_{C_2,1,\mathbb{F}} \oplus S_{C_4,1,\mathbb{F}} \oplus 2S_{V_4,1,\mathbb{F}} \oplus 2S_{V_4,u,\mathbb{F}} \oplus S_{D_8,1,\mathbb{F}} \oplus S_{D_8,1,\mathbb{F}_{-1}}\,.$$

Proof Let (D,e_D) be a maximal b-Brauer pair and for any $P \leqslant D$, let e_P denote the unique block idempotent of $kC_G(P)$ with $(P,e_P) \leqslant (D,e_D)$. Let $\mathcal{F}$ denote the fusion system of b with respect to (D,e_D).

Note that up to isomorphism, there are three fusion systems on D_8. We denote by $\mathcal{F}_{00}$ the inner fusion system; by $\mathcal{F}_{01}$ the non-inner non-simple fusion system; by $\mathcal{F}_{11}$ the simple fusion system. Note that $\mathcal{F}_{00} \cong \mathcal{F}_D(D)$, $\mathcal{F}_{01} \cong \mathcal{F}_D(\mathrm{Sym}(4))$ and $\mathcal{F}_{11} \cong \mathcal{F}_D(PSL(3,2))$.

By [B74], we have $l(kGb)=1$, if $\mathcal{F}\cong\mathcal{F}_{00}$; $l(kGb)=2$, if $\mathcal{F}\cong\mathcal{F}_{01}$; $l(kGb)=3$, if $\mathcal{F}\cong\mathcal{F}_{11}$. This determines the multiplicity of $S_{1,1,\mathbb{F}}$ in all cases.

Let C_2 be a subgroup of D order 2. Up to G-conjugation, we can assume that C_2 is fully $\mathcal{F}$-centralized, and so the block $kC_G(C_2)e_{C_2}$ has a defect group $C_D(C_2)$ which is isomorphic to D or V_4. In both cases, one can show that $l(kC_G(C_2)e_{C_2})=1$. Therefore, Theorem 2.1 implies that the multiplicity of $S_{C_2,1,\mathbb{F}}$ is equal to the number of $\mathcal{F}$-isomorphism classes of objects isomorphic to C_2. Hence

$$\mathrm{Mult}\left(S_{C_2,1,\mathbb{F}},\mathbb{F}T^{\Delta}_{G,b}\right)=\begin{cases}3, & \text{if } \mathcal{F}\cong\mathcal{F}_{00}\\ 2, & \text{if } \mathcal{F}\cong\mathcal{F}_{01}\\ 1, & \text{if } \mathcal{F}\cong\mathcal{F}_{11}\,.\end{cases}$$

Let C_4 be the cyclic subgroup of order 4 of D. The block idempotent $kC_G(C_4)e_{C_4}$ has a central defect group C_4 and so $l(kC_G(C_4)e_{C_4})=1$. Moreover, in all cases one has $\mathrm{Aut}_{\mathcal{F}}(C_4)\cong\mathrm{Aut}(C_4)\cong C_2$. Therefore, Theorem 2.1 implies that

$$\mathrm{Mult}\left(S_{C_4,1,\mathbb{F}},\mathbb{F}T^{\Delta}_{G,b}\right)=\dim_{\mathbb{F}}\mathbb{F}^{C_2}=1 \quad\text{and}\quad \mathrm{Mult}\left(S_{C_4,1,\mathbb{F}_-},\mathbb{F}T^{\Delta}_{G,b}\right)=\dim_{\mathbb{F}}\mathbb{F}_-^{C_2}=0\,.$$

Let X and Y be the subgroups of D isomorphic to V_4. Note that X and Y are not $\mathcal{F}$-conjugate. We use the convention that $\mathrm{Aut}_{\mathcal{F}}(X)\cong\mathrm{Aut}_{\mathcal{F}}(Y)\cong C_2$ if $\mathcal{F}\cong\mathcal{F}_{00}$; $\mathrm{Aut}_{\mathcal{F}}(X)\cong C_2$ and $\mathrm{Aut}_{\mathcal{F}}(Y)\cong\mathrm{Sym}(3)$ if $\mathcal{F}\cong\mathcal{F}_{01}$; $\mathrm{Aut}_{\mathcal{F}}(X)\cong\mathrm{Aut}_{\mathcal{F}}(Y)\cong\mathrm{Sym}(3)$ if $\mathcal{F}\cong\mathcal{F}_{11}$. In all cases, the blocks $kC_G(X)e_X$ and $kC_G(Y)e_Y$ has central defect groups X and Y, respectively, and hence $l(kC_G(X)e_X)=l(kC_G(Y)e_Y)=1$.

Now let $J\in\{X,Y\}$. First assume that $\mathrm{Aut}_{\mathcal{F}}(J)\cong C_2$. Then, one has

$$[N_G(J,e_J)\backslash\mathcal{P}_{(J,e_J)}(V_4,1)/\mathrm{Aut}(V_4)]=[\mathrm{id}]$$

and

$$\mathrm{Aut}(V_4)_{\overline{(J,e_J,\mathrm{id})}}=\{\phi\in\mathrm{Aut}(V_4)\,|\,\phi=i_g, g\in N_G(J,e_J)\}\cong C_2\,.$$

It follows that the $\mathbb{F}$-dimension of

$$\bigoplus_{\pi\in[\mathcal{P}_{(J,e_J)}(V_4)]}\mathbb{F}\mathrm{Proj}(ke_JC_G(J))\otimes_{\mathrm{Aut}(V_4)_{\overline{(J,e_J,\pi)}}}V=\mathbb{F}\otimes_{C_2}V\cong V^{C_2}$$

is equal to one for $V = \mathbb{F}$ and $V = V_2$ and zero for $V = \mathbb{F}_{-1}$. Moreover one has

$$\mathcal{P}_{(J,e_J)}(V_4, u) = \{\phi \in \mathrm{Aut}(V_4) : \phi i_u \phi^{-1} \in \mathrm{Aut}_{\mathcal{F}}(J)\} = \emptyset$$

since $\phi i_u \phi^{-1}$ has order 3.

Next, suppose that $\mathrm{Aut}_{\mathcal{F}}(J) \cong \mathrm{Sym}(3)$. We have

$$[N_G(J, e_J) \backslash \mathcal{P}_{(J,e_J)}(V_4, 1)/\mathrm{Aut}(V_4)] = [\mathrm{id}]$$

and

$$\mathrm{Aut}(V_4)_{\overline{(J,e_J,\mathrm{id})}} = \{\phi \in \mathrm{Aut}(V_4) |\, \phi = i_g, g \in N_G(J, e_J)\} \cong \mathrm{Sym}(3)\,.$$

Therefore, the $\mathbb{F}$-dimension of

$$\bigoplus_{\pi \in [\mathcal{P}^G_{(J,e_J)}(V_4)]} \mathbb{F}\mathrm{Proj}(ke_J C_G(J)) \otimes_{\mathrm{Aut}(V_4)_{\overline{(J,e_J,\pi)}}} V = \mathbb{F} \otimes_{\mathrm{Sym}(3)} V \cong V^{\mathrm{Sym}(3)}$$

is non-zero only for $V = \mathbb{F}$. Moreover,

$$\mathcal{P}_{(J,e_J)}(V_4, u) = \{\phi \in \mathrm{Aut}(V_4) |\, \phi i_u \phi^{-1} \in \mathrm{Aut}_{\mathcal{F}}(J)\} = \mathrm{Aut}(V_4) \cong S_3$$

and

$$[N_G(J, e_J) \backslash \mathcal{P}_{(J,e_J)}(V_4, u)/\mathrm{Aut}(V_4, u)] = [\mathrm{id}]\,.$$

Thus, the $\mathbb{F}$-dimension of

$$\bigoplus_{\pi \in [\mathcal{P}^G_{(J,e_J)}(V_4,u)]} \mathbb{F}\mathrm{Proj}(ke_J C_G(J), u) \otimes_{\mathrm{Aut}(V_4)_{\overline{(J,e_J,\pi)}}} \mathbb{F}$$

is equal to one. These show that the multiplicities of $S_{V_4,1,\mathbb{F}}$, $S_{V_4,1,\mathbb{F}_-}$, $S_{V_4,1,V_2}$ and $S_{V_4,u,\mathbb{F}}$ in $\mathbb{F}T^{\Delta}_{G,b}$ are as claimed.

Finally, since in all cases we have $\mathrm{Aut}_{\mathcal{F}}(D) \cong \mathrm{Inn}(D)$, Lemma 2.4 implies that

$$\mathrm{Mult}\left(S_{D_8,1,\mathbb{F}}, \mathbb{F}T^{\Delta}_{G,b}\right) = \mathrm{Mult}\left(S_{D_8,1,\mathbb{F}_-}, \mathbb{F}T^{\Delta}_{G,b}\right) = 1\,.$$

This completes the proof. $\square$

7 Blocks with $C_2 \times C_2 \times C_2$ defect groups

Let b be a block idempotent of kG with defect groups isomorphic to $D = C_2 \times C_2 \times C_2$. Let E be the inertial quotient of b. Then E has order 1, 3, 7 or 21. By a result of Rouquier in [Ro01] (see also [KKL12, Theorem 21.1]), there is a stable splendid Rickard equivalence, and hence a stable functorial equivalence over $\mathbb{F}$, between b and its Brauer correspondent block c. Since E determines the number of simple modules in the block, it follows from [BY223, Theorem 1.2(i)] that there is a functorial equivalence between b and c. So we can assume that D is normal in G. Therefore, b has a source algebra $k(D \rtimes E)$. This shows that E determines the functorial equivalence class over $\mathbb{F}$ of b. Therefore we have the following.

7.1 Theorem *Let b be a block idempotent of kG with defect group isomorphic to $C_2 \times C_2 \times C_2$ and let E be the inertial quotient of b. Then one of the following occurs:*

(i) $|E| = 1$ *and* (G, b) *is functorially equivalent over* $\mathbb{F}$ *to* $(C_2 \times C_2 \times C_2, 1)$.

(ii) $|E| = 3$ *and* (G, b) *is functorially equivalent over* $\mathbb{F}$ *to* $(A_4 \times C_2, 1)$.

(iii) $|E| = 7$ *and* (G, b) *is functorially equivalent over* $\mathbb{F}$ *to* $(SL_2(8), b_0(SL_2(8)))$.

(iv) $|E| = 21$ *and* (G, b) *is functorially equivalent over* $\mathbb{F}$ *to* $(J_1, b_0(J_1))$.

In particular, for blocks with $C_2 \times C_2 \times C_2$ defect groups, functorial equivalence classes over $\mathbb{F}$ coincide with isotypy classes.

8 Blocks with $C_2 \times C_4$ defect groups

For completeness, we consider the blocks with defect groups $C_2 \times C_4$. Since $\mathrm{Aut}(C_2 \times C_4) \cong D_8$, the functors $S_{C_2\times C_4,1,V}$ are the only simple functors with parametrizing set (L, u, V) with $L \cong C_2 \times C_4$, where $V \in \{\mathbb{F}, \mathbb{F}_1, \mathbb{F}_2, \mathbb{F}_3, V_2\}$ is a simple $\mathbb{F}D_8$-module.

8.1 Theorem *Let b be a block idempotent of kG with defect groups isomorphic to $C_2 \times C_4$. Then (G, b) is functorially equivalent over $\mathbb{F}$ to $(C_2 \times C_4, 1)$. Moreover, one has*

$$\mathrm{Mult}\left(S_{1,1,\mathbb{F}}, \mathbb{F}T^{\Delta}_{G,b}\right) = 1, \quad \mathrm{Mult}\left(S_{C_2,1,\mathbb{F}}, \mathbb{F}T^{\Delta}_{G,b}\right) = 3,$$

$$\mathrm{Mult}\left(S_{C_4,1,V}, \mathbb{F}T^{\Delta}_{G,b}\right) = 2 \quad \textit{for } V \in \{\mathbb{F}, \mathbb{F}_-\},$$

$$\mathrm{Mult}\left(S_{V_4,1,V}, \mathbb{F}T^{\Delta}_{G,b}\right) = \dim_{\mathbb{F}} V \quad \textit{for } V \in \{\mathbb{F}, \mathbb{F}_-, V_2\},$$

$$\mathrm{Mult}\left(S_{C_2\times C_4,1,V}, \mathbb{F}T^{\Delta}_{G,b}\right) = \dim_{\mathbb{F}} V \quad \textit{for } V \in \{\mathbb{F}, \mathbb{F}_1, \mathbb{F}_2, \mathbb{F}_3, V_2\}.$$

Proof Since $C_2 \times C_4$ has no automorphism of odd order, the block kGb is nilpotent and hence by [BY22, Theorem 9.2], (G, b) is functorially equivalent over $\mathbb{F}$ to $(C_2 \times C_4, 1)$. One can find the multiplicities using Theorem 2.2. □

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